Matrices + Your Calculator

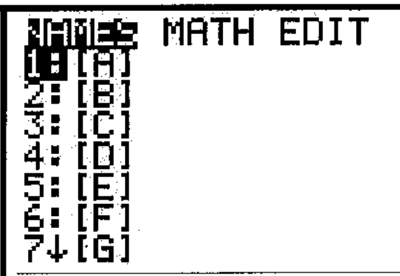
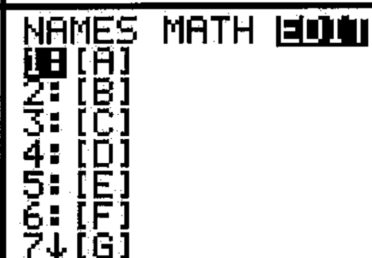
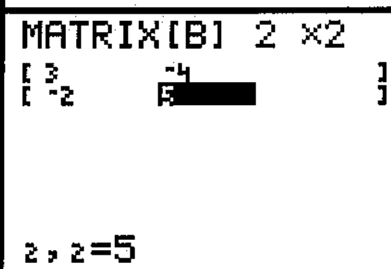
Please do/practice these steps

A. Inputting a Matrix

Consider the following two matrices A and B.

$$A = \begin{bmatrix} -1 & 5 \\ 3 & 8 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 3 & -4 \\ -2 & 5 \end{bmatrix}.$$

We are going to enter Matrix A and Matrix B in our calculator. Below are the step-by-step instructions.

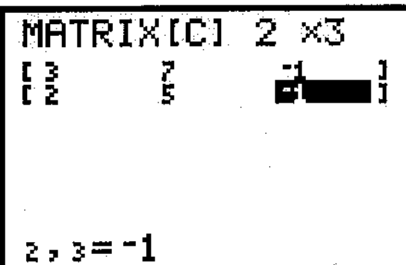
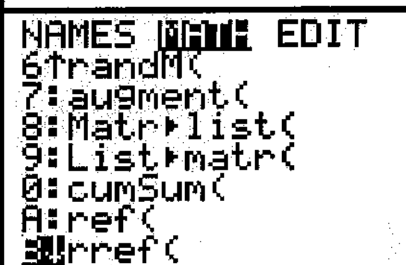
On your calculator, look at the far left column of buttons. There is a button labeled x^{-1}. Above this button is the command entitled MATRIX (or MATRX). Press the 2nd button and then press the x^{-1} button to gain access to the Matrix menus. After pressing these buttons, your screen should appear as shown.	
At the top of your screen, you'll see the words NAMES, MATH and EDIT. These are names of menus. Use your Right/Left arrow button to highlight the EDIT menu. Highlight the line 1: [A] and press ENTER.	
Enter the dimensions of Matrix A: number of rows then press ENTER; number of columns then press ENTER.	
Enter the elements of Matrix A by highlighting the appropriate cell, typing the number and pressing ENTER. You have now completely entered Matrix A. Notice the cell location at the bottom left corner of your screen.	
Now you will enter Matrix B by performing the same steps.	
We are ready to use these matrices. We'll first return to our Home Screen by using the QUIT command. This is above the MODE button near the top of your calculator. Press 2nd and MODE to access the QUIT command.	

8. Matrices as Systems of Equations

Matrices are also used to denote a system of equations. The below system of equations can be written in matrix form:

$$\begin{aligned} 3x+7y&=-1 \\ 2x+5y&=-1 \end{aligned} \longrightarrow \left[\begin{array}{cc|c} 3 & 7 & -1 \\ 2 & 5 & -1 \end{array}\right]$$

We're interested in solving this system of equations (i.e. determining the values of x and y that make both equations true simultaneously). Below are the instructions to have your calculator solve the system of equations in matrix form.

First, we'll input the matrix representing the system of equations in our calculator in the same manner as discussed above. We'll call this matrix [C]. For purposes of the calculator, we ignore the vertical line between columns 2 and 3 in our original matrix.	
We return to our Home Screen by using the QUIT command.	
Now, press MATRIX and Right Arrow to the MATH Menu. Down Arrow to command B: rref.	
Press ENTER to select this command. Your calculator should take you back to the Home Screen with rref(' on the command line. Press MATRIX and select Matrix C. Press the Right Parenthesis button, and then press ENTER. Your solution matrix will appear on your screen. This matrix indicates that the solution is x = 2 and y = -1.	

C. Performing Operations With These Matrices

This section will demonstrate the calculator steps for addition. Other operations can be performed in similar manner.

We can use the calculator to perform operations using these matrices. For instance, we may want to determine A+B. Access the MATRIX command. You should now see the screen displaying the dimensions of Matrices A and B.	NAMES MATH EDIT 1:[A] 2×2 2:[B] 2×2 3:[C] 4:[D] 5:[E] 6:[F] 7↓[G]
With Matrix A highlighted, press ENTER. Now press the + button. Press MATRIX again and this time select Matrix B. Once you have [A] + [B] on your screen, press ENTER to get the answer. Your screen should now look as to the right.	[A]+[B] [[2 1] [1 13]]

D. Deleting Matrices

We want to be able to delete a matrix when we are finished using it. Below are the instructions to delete matrices.

Above the blue + key is a command labeled MEM. Press 2nd and the + key to access the MEM menu. Then highlight 2: Mem Mgmt/Del … command and press ENTER.	MEMORY 1:About 2:Mem Mgmt/Del… 3:Clear Entries 4:ClrAllLists 5:Archive 6:UnArchive 7↓Reset…
Highlight 5: Matrix and press ENTER.	RAM FREE 19358 ARC FREE 8582 1:All… 2:Real… 3:Complex… 4:List… 5:Matrix… 6↓Y-Vars…
We now see a list containing the three matrices we created. We're going to delete each of these. Using your up/down arrow, move the pointer to [A] and press the DEL key (found immediately to the right of the MODE key). The entry for matrix A will be deleted. You can continue in similar manner for matrices B and C.	RAM FREE 19358 ARC FREE 8582 ▸ [A] 47 [B] 47 [C] 65

A MORE IN DEPTH LOOK AT

SOLVING SYSTEMS OF EQUATIONS WITH MATRICES ON THE TI-83

Example Solve the system of equations:

$$\begin{cases} 2x+3y-z=9 \\ x-y+4z=6 \\ 2y+3z=12 \end{cases}$$

A system of linear equations can be written in a more abbreviated form using matrices:

$$\begin{cases} 2x+3y-z=9 \\ x-y+4z=6 \\ 2y+3z=12 \end{cases} \Rightarrow \begin{bmatrix} 2 & 3 & -1 & 9 \\ 1 & -1 & 4 & 6 \\ 0 & 2 & 3 & 12 \end{bmatrix}$$

Then, using elementary row operations (see Appendix E for the theory of row operations and matrices) we can transform this *initial* matrix into a *Reduced Row Echelon Form* matrix:

$$\begin{bmatrix} 2 & 3 & -1 & 9 \\ 1 & -1 & 4 & 6 \\ 0 & 2 & 3 & 12 \end{bmatrix} \overset{RREF}{\Rightarrow} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

Each row of this *final*, or *reduced*, matrix can be rewritten back into their equivalent equations

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \end{bmatrix} \Rightarrow \begin{cases} x+0y+0z=1 \\ 0x+y+0z=3 \\ 0x+0y+z=2 \end{cases}$$

or, more simply: $\begin{cases} x=1 \\ y=3 \\ z=2 \end{cases}$. Voila, our solution! ■

* A couple special reduced matrices to watch out for:

A) Consider the reduced matrix $\begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$. Notice, the last row has an equivalent equation of $0x+0y+0z=1$ or $0=1$. Obviously this is not possible, meaning the original system of equations **does not have a solution.**

B) Consider the reduced matrix $\begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & -3 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

This time the last row gives a valid equation: $0=0$. What this type of matrix tells us is that there are actually an infinite number of solutions. To find the solutions, write the first two rows as equations

$$\begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & -3 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{cases} x+2z=3 \\ y-3z=5 \end{cases}$$

Let $z=t$, and solve for x and y respectively. This will give the solution

$$\begin{cases} x=-2t+3 \\ y=3t+5 \\ z=t \end{cases}$$

We can choose any value for t and obtain an ordered triple, which will solve the original system of equations. ■

* USING MATRICES WITH THE TI-83

The first step in solving a system of equations is to write the system in matrix form, and store the matrix into a matrix variable.

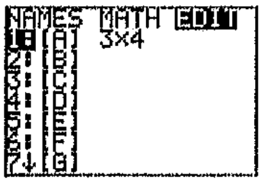
Example: Solve $\begin{cases} 3x+6y+z=7 \\ x-3y-2z=3 \\ 2x+y+z=12 \end{cases}$

#1

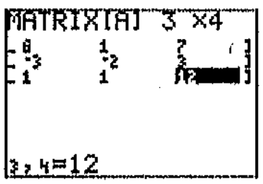
1. Write in matrix form:

$$\begin{cases} 3x+6y+z=7 \\ x-3y-2z=3 \\ 2x+y+z=12 \end{cases} \Rightarrow \begin{bmatrix} 3 & 6 & 1 & 7 \\ 1 & -3 & -2 & 3 \\ 2 & 1 & 1 & 12 \end{bmatrix}$$

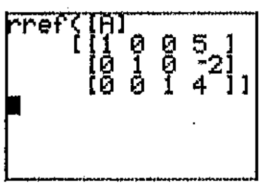
2. Enter the **matrix editor** by pressing **[2nd] [MTRX]** and move over to the **EDIT** menu.



3. Enter the size of the matrix (3x4) and enter each element of the matrix.



4. When all the elements are entered press **[2nd] [QUIT]**
5. Next, we need to find the **RREF** command and instruct the calculator to "rref" matrix [A]. Press: **[2nd] [MTRX]**, move over to the **MATH** menu and scroll down and select option **B:rref(**. Press **[2nd] [MTRX]** and choose **1:[A]** (or the matrix variable you used), and press **[ENTER]** twice. This should produce a reduced matrix.



$$\begin{bmatrix} 3 & 6 & 1 & 7 \\ 1 & -3 & -2 & 3 \\ 2 & 1 & 1 & 12 \end{bmatrix} \overset{RREF}{\Rightarrow} \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

Our solution to the original system is:

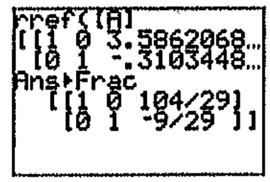
$$\begin{cases} x = 5 \\ y = -2 . \\ z = 4 \end{cases}$$

Example: Solve the system of equations:

#2

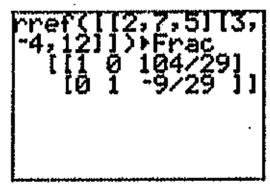
$$\begin{cases} 2x + 7y = 5 \\ 3x - 4y = 12 \end{cases}.$$

Note: this system will result in a 2x3 matrix. Also, you should have discovered that the solution consists of fairly ugly decimal values. These can be cleaned up by using the ▸ **FRAC** command found under the **[MATH]** menu:



The solution is the point $\left(\dfrac{104}{29}, \dfrac{-9}{29}\right)$.

Side note: These problems can be done in a single step without using a matrix variable or the editor:



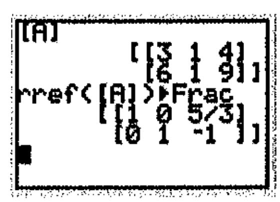
#3

Example: Use matrices to find the equation of the line that passes through the points $(3,4)$ and $(6,9)$. Write in slope-intercept form $y = mx + b$.

Begin my substituting each (x, y) point into the slope-intercept form:

$$\begin{cases} 4 = m \cdot 3 + b \\ 9 = m \cdot 6 + b \end{cases} \Rightarrow \begin{cases} 3m + b = 4 \\ 6m + b = 9 \end{cases}$$

Write in matrix form and "rref".



$$\begin{bmatrix} 3 & 1 & 4 \\ 6 & 1 & 9 \end{bmatrix} \overset{RREF}{\Rightarrow} \begin{array}{c} \begin{bmatrix} 1 & 0 & 5/3 \\ 0 & 1 & -1 \end{bmatrix} \end{array} \begin{array}{l} \leftarrow m \\ \leftarrow b \end{array}$$

The equation of the line is: $y = \dfrac{5}{3}x - 1$.